Further understanding of the non-$D\bar{D}$ decays of $\psi(3770)$

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We provide details of the study of $\psi(3770)$ non-$D\bar{D}$ decays into VP, where V and P denote light vector meson and pseudoscalar meson, respectively. We find that the electromagnetic (EM) interaction plays little role in these processes, while the strong interaction dominates. The strong interaction can be separated into two parts, i.e. the short-distance part probing the wave function at origin and the long-distance part reflecting the soft gluon exchanged dynamics. The long-distance part is thus described by the intermediate charmed meson loops. We show that the transition of $\psi(3770) \to VP$ can be related to $\psi(3686) \to VP$ such that the parameters in our model can be constrained by comparing the different parts in $\psi(3770) \to VP$ to those in $\psi(3686) \to VP$. Our quantitative results confirm the findings of [Zhang *et al.*, Phys. Rev. Lett. 102, 172001 (2009)] that the OZI-rule-evading long-distance strong interaction via the IML plays an important role in $\psi(3770)$ decays, and could be a key towards a full understanding of the mysterious $\psi(3770)$ non-$D\bar{D}$ decay mechanism.

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I. INTRODUCTION

The $\psi(3770)$ is the lowest mass charmonium resonance above the open charm pair $D\bar{D}$ production threshold. Traditional theories expect that it decays almost entirely into pure $D\bar{D}$ pairs without the so-called Okubo-Zweig-Iizuka (OZI) rule [1] suppression. An interesting and nontrivial question here is whether the $\psi(3770)$ decay is totally saturated by $D\bar{D}$, or whether there exist significant non-$D\bar{D}$ decay channels. CLEO measured the exclusive cross section $\sigma(e^+e^- \to D^0\bar{D}^0) = (3.66 \pm 0.03 \pm 0.06)$ nb, $\sigma(e^+e^- \to D^+D^-) = (2.91 \pm 0.03 \pm 0.05)$ nb at the center of mass energy 3774 ± 1MeV [2, 3] and the inclusive cross section $\sigma(e^+e^- \to \psi(3770) \to \text{non-}D\bar{D}) = -0.01 \pm 0.08^{+0.41}_{-0.30}$ nb [4]. These results lead to $\text{BR}(\psi(3770) \to D\bar{D}) = (103.0 \pm 1.4^{+5.1}_{-6.8})\%$, of which the lower bound suggests the maximum non-$D\bar{D}$ branching ratio is about 6.8%.

BES earlier reported two results based on different analysis methods: $\text{BR}(\psi(3770) \to \text{non-}D\bar{D}) = (14.5 \pm 1.7 \pm 5.8)\%$ [5] and $\text{BR}(\psi(3770) \to \text{non-}D\bar{D}) = (16.4 \pm 7.3 \pm 4.2)\%$ [2]. With the first direct measurement on the non-$D\bar{D}$ decay, BES gives $\sigma(e^+e^- \to \psi(3770) \to \text{non-}D\bar{D}) = (0.95 \pm 0.35 \pm 0.29)$ nb and $\text{BR}(\psi(3770) \to \text{non-}D\bar{D}) = (13.4 \pm 5.0 \pm 3.6)\%$ [6]. Evidently, the two collaborations give very different results of the non-$D\bar{D}$ decay of $\psi(3770)$. Meanwhile, various exclusive decay channels have been investigated by BES [7–11] and CLEO [12–14] in order to search for further non-$D\bar{D}$ decay modes of $\psi(3770)$.

In Ref. [15], several non-$D\bar{D}$ hadronic decay branching ratios are listed, i.e. $\psi(3770) \to J/\psi\pi^+\pi^-$, $J/\psi\pi^0\pi^0$, $J/\psi\eta$ and $\phi\eta$, while tens of other channels have only experimental upper limits due to the poor statistics. These results are within the range of theoretical predictions based on the QCD multipole

expansion for hadronic transitions [16]. With the total width of 27.3 ± 1.0 MeV for $\psi(3770)$ [15], the width of all hadronic transitions is about $100 - 150$ KeV. Another kind of non-$D\bar{D}$ decays of $\psi(3770)$ are the E1 transitions $\psi(3770) \to \gamma\chi_{c0}$ and $\gamma\chi_{c1}$, their branching ratios are measured to be $(7.3 \pm 0.9) \times 10^{-3}$, $(2.9 \pm 0.6) \times 10^{-3}$, respectively [15], while only an upper limit is given to $\psi(3770) \to \gamma\chi_{c2}$. The sum of those channels, however, is far from clarifying the mysterious situation of the $\psi(3770)$ non-$D\bar{D}$ decays. It hence stimulates intensive experimental and theoretical efforts [16–23] on understanding the nature of $\psi(3770)$ and its strong and radiative transition dynamics.

Theoretically, Kuang and Yan [16] calculated the transition $\psi(3770) \to J/\psi\pi\pi$ with the QCD multipole expansion, which indicated small exclusive non-$D\bar{D}$ decays width at leading order. Recently, He, Fan and Chao [23] calculate the light hadron decays of $\psi(3770)$ based on nonrelativistic QCD factorization at next-to-leading order (NLO) in α_s and leading order in v^2, which gives $\Gamma[\psi(3770) \to \text{light hadrons}] = 467^{-187}_{+338}$ keV. Although the NLO contributions are found important, it has also been pointed out in Ref. [23] that any significant non-$D\bar{D}$ branching ratios up to a few percent would be likely due to non-perturbative mechanisms instead of QCD higher order contributions. Since the mass of $\psi(3770)$ is close to the $D\bar{D}$ threshold, a natural mechanism involving non-perturbative mechanisms is the $D\bar{D}$ final state interaction. Namely, the rescattering of the $D\bar{D}$ as intermediate states can contribute to final state non-$D\bar{D}$ decay channels via charmed meson loops. Such a mechanism has been quantified in Ref. [24] for $\psi(3770) \to VP$, where the intermediate meson loops (IML) are extended to include $D\bar{D}^* + c.c.$ contributions. Since the mass threshold of $D\bar{D}^* + c.c.$ is higher than the mass of $\psi(3770)$, it should be noted that only the intermediate $D\bar{D}$ can contribute to the absorptive part of the non-$D\bar{D}$ transition amplitude, while other IML will contribute to the dispersive part. Taking into account that the $\psi(3770)$ is coupled to the $D\bar{D}$ pair by a P wave, we also note that model-dependence of the IML calculations will become inevitable when evaluate the dispersive amplitudes. In Ref. [25] the same mechanism is also investigated for $\psi(3770) \to J/\psi\eta$, $\rho\pi$, $\omega\eta$ and K^*K, which is consistent with the result of Ref. [24].

The impact of the IML as an important non-perturbative mechanism has been extensively studied in charmonium decays during the past few years. It indicates that since the charm quark is not heavy enough the decay of charmonium states would be sensitive to the interferences between the perturbative and non-perturbative mechanisms in the charmonium energy region. With more and more data from Belle, BaBar, CLEO and BES-III, it is broadly recognized that the intermediate hadron loops can be closely related to a lot of non-perturbative phenomena observed in experiment, e.g. apparent OZI-rule violations [24–39], helicity selection rule violations [26–28], and isospin symmetry breakings in charmonium decays [40, 41]. In this work, we present a more detailed analysis of the $\psi(3770)$ non-$D\bar{D}$ decays into a pair of vector and pseudoscalar mesons. In particular, we try to constrain the model parameters such that a quantitative evaluation of the decay branching ratios of $\psi(3770) \to VP$ can be achieved.

This paper is organized as below. In Sec. II we present the framework of our calculation. Section III is devoted to numerical results and discussions. A brief summary is given in the last section.

II. THE FRAMEWORK

The IML contributions in $\psi(3770)$ can be related to the similar processes of $\psi(3686) \to VP$ as discussed in Ref. [33]. This is understandable since both $\psi(3770)$ and $\psi(3686)$ are located in the vicinity of the $D\bar{D}$ threshold. Thus, their decays would experience similar effects from the $D\bar{D}$ threshold. Such a conjecture allows us to correlate these two states together such that some of the parameters can be fixed for both cases. Meanwhile, we stress some differences between these two decays, i.e. $\psi(3770)$ and $\psi(3686) \to VP$. For $\psi(3770)$ the intermediate $D\bar{D}$ meson loop can contribute to both the absorptive and dispersive part of the transition amplitude, while for $\psi(3686)$ the IML will only contribute to the dispersive part since $\psi(3686)$ is below the $D\bar{D}$ threshold.

Benefiting from the unique antisymmetric tensor structure for the VVP coupling, we can decompose the transition amplitude into three parts, i.e. the EM interaction, short-distance strong interaction and long-distance strong interaction.
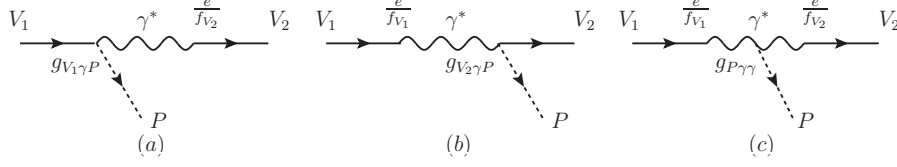
FIG. 1: The tree-level Feynman diagrams of EM transitions in $\psi(3770) \to VP$.

A. The electromagnetism contribution

As shown in Fig. 1, the tree level amplitudes of the EM part can be described by the vector meson dominance (VMD),

$$
\begin{aligned}
\mathcal{M}_{EM} &= \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c \\
&= \left(\frac{e}{f_{V_2}} \frac{g_{V_1\gamma P}}{M_{V_1}} \mathcal{F}_a + \frac{e}{f_{V_1}} \frac{g_{V_2\gamma P}}{M_{V_2}} \mathcal{F}_b + \frac{e^2}{f_{V_1} f_{V_2}} \frac{g_{P\gamma\gamma}}{M_P} \mathcal{F}_c \right) \epsilon_{\mu\nu\alpha\beta} p^\mu \epsilon(p)^\nu k^\alpha \epsilon(k)^\beta,
\end{aligned}
\tag{1}
$$

where $p(k)$ is the four momentum of the initial vector charmonium (final light vector), and $\epsilon(p)$ ($\epsilon(k)$) is its corresponding polarization vector. The effective couplings in the amplitudes can be found in Table. I-III of Ref. [33]. The effective coupling $e/f_{\psi(3770)} = 5.4 \times 10^{-3}$ [28] is used here. Meanwhile, the upper limits of the branching ratios $\mathrm{BR}(\psi(3770) \to \gamma\pi^0) < 2 \times 10^{-4}$, $\mathrm{BR}(\psi(3770) \to \gamma\eta) < 1.5 \times 10^{-4}$ and $\mathrm{BR}(\psi(3770) \to \gamma\eta') < 1.8 \times 10^{-4}$ are used to extract the effective couplings $g_{\gamma\psi(3770)\pi^0} = 4.38 \times 10^{-4}$, $g_{\gamma\psi(3770)\eta} = 3.49 \times 10^{-4}$ and $g_{\gamma\psi(3770)\eta'} = 4.8 \times 10^{-4}$. Considering the off-shell photon, a monopole form factor $\mathcal{F}(q^2) = \Lambda_{EM}^2/(\Lambda_{EM}^2 - q^2)$ is included. Since the mass difference between $\psi(3770)$ and $\psi(3686)$ is rather small, we adopt the same cut off parameter $\Lambda_{EM} = 0.542$ GeV as in $\psi(3686)$ decays [33] to evaluate the EM contribution. Namely, the EM contribution can be constrained by taking into account the $\psi(3686)$ transitions.

B. The short-distance strong interaction

As shown in Fig. 2, the short-distance strong interaction is $c\bar{c}$ annihilation into three hard gluons. We use the simple parameterized scheme [33] to describe this part. The amplitudes of the VP channels can be written as

$$
\begin{aligned}
\mathcal{M}_S(\rho^0\pi^0) &= \mathcal{M}_S(\rho^+\pi^-) = \mathcal{M}_S(\rho^-\pi^+) = g\mathcal{F}(\mathbf{P}) , \\
\mathcal{M}_S(K^{*+}K^-) &= \mathcal{M}_S(K^{*-}K^+) = \mathcal{M}_S(K^{*0}\bar{K}^0) = \mathcal{M}_S(\bar{K}^{*0}K^0) = g\xi\mathcal{F}(\mathbf{P}) , \\
\mathcal{M}_S(\omega\eta) &= X_\eta g(1+2r)\mathcal{F}(\mathbf{P}) + Y_\eta \sqrt{2}\xi r g\mathcal{F}(\mathbf{P}) , \\
\mathcal{M}_S(\omega\eta') &= X_{\eta'} g(1+2r)\mathcal{F}(\mathbf{P}) + Y_{\eta'} \sqrt{2}\xi r g\mathcal{F}(\mathbf{P}) , \\
\mathcal{M}_S(\phi\eta) &= X_\eta \sqrt{2}\xi r g\mathcal{F}(\mathbf{P}) + Y_\eta g(1+r)\xi^2\mathcal{F}(\mathbf{P}) , \\
\mathcal{M}_S(\phi\eta') &= X_{\eta'} \sqrt{2}\xi r g\mathcal{F}(\mathbf{P}) + Y_{\eta'} g(1+r)\xi^2\mathcal{F}(\mathbf{P}),
\end{aligned}
\tag{2}
$$

where g, ξ and r are the transition strength of the singly disconnected OZI (SOZI) process, SU(3) breaking parameter and doubly disconnected OZI (DOZI) process parameter [33]. $\mathcal{F}(\mathbf{P}) \equiv |\mathbf{P}|^l \exp(-\mathbf{P}^2/16\beta^2)$ (with $\beta = 0.5$ GeV) is the exponential form factor which is obtained from the chiral constitute quark model reflecting the size effect of the initial and final particles.
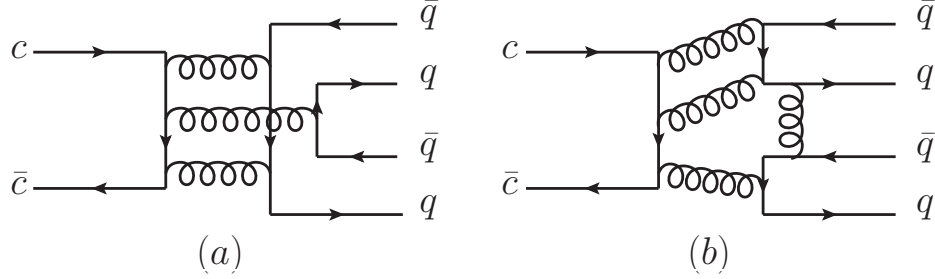
FIG. 2: Schematic diagrams for the short-distance strong transitions in $\psi(3770) \to VP$. Diagram (a) illustrates the SOZI process, while (b) is for the DOZI one. In both cases, c and $\bar{c}$ annihilate at the origin of the wavefunction.

The η-η' mixing is considered in a standard way,

$$
\begin{aligned}
\eta &= \cos \alpha_P |n\bar{n}\rangle - \sin \alpha_P |s\bar{s}\rangle, \\
\eta' &= \sin \alpha_P |n\bar{n}\rangle + \cos \alpha_P |s\bar{s}\rangle,
\end{aligned}
\tag{3}
$$

where $|n\bar{n}\rangle \equiv |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$, and the mixing angle $\alpha_P = \theta_P + \arctan(\sqrt{2})$ with $\theta_P \simeq -24.6°$ or $\sim -11.5°$ for linear or quadratic mass relations, respectively [15]. We adopt $\theta_P = -22°$ [33]. Here, $X_\eta = Y_{\eta'} = \cos \alpha_P$ and $-Y_\eta = X_{\eta'} = \sin \alpha_P$.

Again, since the mass difference between $\psi(3770)$ and $\psi(3686)$ is not large, the SU(3) breaking effect and DOZI parameter can be taken the same as those determined in the $\psi(3686)$ decays, i.e. $\xi = 0.92$, $r = -0.097$. Meanwhile, the wave function at origin can be extracted from the lepton pair decay width. The branching ratios $BR(\psi(3686) \to e^+ e^-) = 7.72 \times 10^{-3}$ and $BR(\psi(3770) \to e^+ e^-) = 9.7 \times 10^{-6}$ gives the ratio of the wave function at origin $\frac{\psi(3770)(0)}{\psi(3686)(0)} = 0.35$, implying $g_{\psi(3770)} \simeq 0.35 \times g_{\psi(3686)} = 0.35 \times 1.25 \times 10^{-2} = 4.38 \times 10^{-3}$. Under this circumstance, the short-distance strong contribution is also determined.

C. The long-distance strong interaction

Similar to Ref. [24], the long-distance contributions are considered by the IML as illustrated in Fig. 3. As studied in Ref. [24], the s-channel process via $2S - 1D$ mixing does not contribute significantly to the amplitude. Therefore, only the t-channel transitions are studied in the evaluation. In addition, an improvement here is that the form factor is introduced in line with the study of the $\psi(3686)$ decays [33] such that a correlation between these two states can be established and provide more stringent constraints on the underlying mechanisms. Since the non-local effects cannot be well controlled in the ELA framework, such a correlation would be crucially useful for quantifying the IML as a possible non-perturbative mechanism in charmonium decays.

The couplings between an S-wave charmonium and charmed mesons are given by the effective Lagrangian based on heavy quark symmetry [42, 43],

$$
\mathcal{L}_2 = ig_2 \mathrm{Tr}[R_{c\bar{c}} \bar{H}_{2i} \gamma^\mu \overleftrightarrow{\partial}_\mu \bar{H}_{1i}] + H.c.,
\tag{4}
$$

where the S-wave vector and pseudoscalar charmonium states are expressed as

$$
R_{c\bar{c}} = \left(\frac{1 + \slashed{v}}{2} \right) (\psi^\mu \gamma_\mu - \eta_c \gamma_5) \left(\frac{1 - \slashed{v}}{2} \right).
\tag{5}
$$

The charmed and anti-charmed meson triplet are

$$H_{1i} = \left(\frac{1+\slashed{v}}{2}\right)[\mathcal{D}_i^{*\mu}\gamma_\mu - \mathcal{D}_i\gamma_5], \tag{6}$$

$$H_{2i} = [\bar{\mathcal{D}}_i^{*\mu}\gamma_\mu - \bar{\mathcal{D}}_i\gamma_5]\left(\frac{1-\slashed{v}}{2}\right), \tag{7}$$

where $\mathcal{D}$ and $\mathcal{D}^*$ are pseudoscalar ((D^0, D^+, D_s^+)) and vector charmed mesons ($(D^{*0}, D^{*+}, D_s^{*+})$), respectively.

Explicitly, the Lagrangian for $\psi(3770)$ couplings to D and D^* becomes

$$\begin{aligned}
\mathcal{L}_\psi &= ig_{\psi\mathcal{D}^*\mathcal{D}^*}(-\psi^\mu\mathcal{D}^{*\nu}\overset{\leftrightarrow}{\partial}_\mu\mathcal{D}_\nu^{*\dagger} + \psi^\mu\mathcal{D}^{*\nu}\partial_\nu\mathcal{D}_\mu^{*\dagger} - \psi_\mu\partial_\nu\mathcal{D}^{*\mu}\mathcal{D}^{*\nu\dagger}) \\
&+ ig_{\psi\mathcal{D}\mathcal{D}}\psi_\mu(\partial^\mu\mathcal{D}\mathcal{D}^\dagger - \mathcal{D}\partial^\mu\mathcal{D}^\dagger) + g_{\psi\mathcal{D}^*\mathcal{D}}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\psi_\nu(\mathcal{D}_\alpha^*\overset{\leftrightarrow}{\partial}_\beta\mathcal{D}^\dagger - \mathcal{D}\overset{\leftrightarrow}{\partial}_\beta\mathcal{D}_\alpha^{*\dagger}),
\end{aligned} \tag{8}$$

with

$$g_{\psi DD} = g_{\psi D^*D^*}, \quad g_{\psi D^*D} = \frac{g_{\psi DD}}{\sqrt{m_{D^*}m_D}}. \tag{9}$$

The coupling $g_{\psi(3770)D\bar{D}}$ is extracted with the help of the experimental data [15],

$$\Gamma_{\psi(3770)\to D\bar{D}} = \frac{g_{\psi(3770)D\bar{D}}^2|\vec{p}|^3}{6\pi M_{\psi(3770)}^2}, \tag{10}$$

where $|\vec{p}|$ is the three momentum of final D-meson. The branching ratios for $\psi(3770) \to D^+D^-$ and $D^0\bar{D}^0$ are slightly different, which gives $g_{\psi(3770)D^+D^-} = 13.55$ and $g_{\psi(3770)D^0\bar{D}^0} = 12.69$. The averaged value 13.12 is used in our calculation.

The Lagrangians relevant to the light vector and pseudoscalar mesons are,

$$\begin{aligned}
\mathcal{L} &= -ig_{D^*DP}(D^i\partial^\mu P_{ij}D_\mu^{*j\dagger} - D_\mu^{*i}\partial^\mu P_{ij}D^{j\dagger}) + \frac{1}{2}g_{D^*D^*P}\varepsilon_{\mu\nu\alpha\beta}D_i^{*\mu}\partial^\nu P^{ij}\overset{\leftrightarrow}{\partial}{}^\alpha D_j^{*\beta\dagger} \\
&- ig_{DDV}D_i^\dagger\overset{\leftrightarrow}{\partial}_\mu D^j(V^\mu)_j^i - 2f_{D^*DV}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu V^\nu)_j^i(D_i^\dagger\overset{\leftrightarrow}{\partial}{}^\alpha D^{*\beta j} - D_i^{*\beta\dagger}\overset{\leftrightarrow}{\partial}{}^\alpha D^j) \\
&+ ig_{D^*D^*V}D_i^{*\nu\dagger}\overset{\leftrightarrow}{\partial}_\mu D_\nu^{*j}(V^\mu)_j^i + 4if_{D^*D^*V}D_{i\mu}^{*\dagger}(\partial^\mu V^\nu - \partial^\nu V^\mu)_j^i D_\nu^{*j},
\end{aligned} \tag{11}$$

with the convention $\varepsilon_{0123} = 1$, where P and V_μ are 3×3 matrices for the nonet pseudoscalar and vector mesons, respectively,

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta\cos\alpha_P + \eta'\sin\alpha_P}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta\cos\alpha_P + \eta'\sin\alpha_P}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\eta\sin\alpha_P + \eta'\cos\alpha_P \end{pmatrix}, \tag{12}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{13}$$

In the chiral and heavy quark limits, the charmed meson couplings to light meson have the following relationship [42],

$$g_{D^*D^*\pi} = \frac{g_{D^*D\pi}}{\sqrt{m_D m_{D^*}}} = \frac{2}{f_\pi}g, \quad g_{DDV} = g_{D^*D^*V} = \frac{\beta g_V}{\sqrt{2}}, \quad f_{D^*DV} = \frac{f_{D^*D^*V}}{m_{D^*}} = \frac{\lambda g_V}{\sqrt{2}}, \tag{14}$$

where $f_\pi = 132$ MeV is the pion decay constant, the parameters g_V is given by $g_V = m_\rho/f_\pi$ [42]. We take $\lambda = 0.56\,\text{GeV}^{-1}$ and $g = 0.59$ [44] in the calculation.
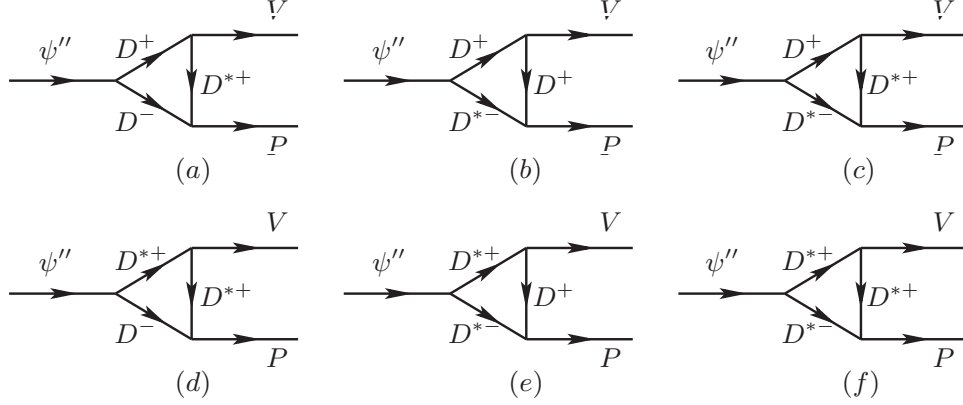
FIG. 3: Diagrams of charged intermediate meson loops contributing to $\psi(3770) \to VP$, where V and P denote light vector and pseudoscalar mesons, respectively. Similar diagrams with the neutral and strange charmed meson loops are implicated.

The loop transition amplitudes in Fig. 3 can be expressed in a general form in the ELA as follows:

$$M_{fi} = \int \frac{d^4 q_2}{(2\pi)^4} \sum_{D^* \text{ pol.}} \frac{V_1 V_2 V_3}{a_1 a_2 a_3} \prod_i \mathcal{F}_i(m_i, q_i^2) \qquad (15)$$

where V_i ($i = 1, 2, 3$) are the vertex functions; $a_i = q_i^2 - m_i^2$ ($i = 1, 2, 3$) are the denominators of the intermediate meson propagators. We adopt the tri-monopole form factor [33], $\prod_i \mathcal{F}_i(m_i, q_i^2)$, which is a product of off-shell monopole form factors for each internal mesons, i.e.

$$\prod_i \mathcal{F}_i(m_i, q_i^2) \equiv \mathcal{F}_1(m_1, q_1^2)\mathcal{F}_2(m_2, q_2^2)\mathcal{F}_3(m_3, q_3^2) \ , \qquad (16)$$

where

$$\mathcal{F}_i(m_i, q_i^2) \equiv \frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - q_i^2}, \qquad (17)$$

with $\Lambda_i \equiv m_i + \alpha \Lambda_{QCD}$ and the QCD energy scale $\Lambda_{QCD} = 220$ MeV. This form factor will take into account the non-local effects of the vertex functions and kill the loop divergence in the integrals.

In principle, one should include all the possible intermediate-meson-exchange loops in the calculation. In reality, the breakdown of the local quark-hadron duality allows us to pick up the leading contributions as a reasonable approximation [45, 46]. Also, intermediate states involving flavor changes turn out to be strongly suppressed. One reason is because of the large virtualities involved. The other is because of the OZI rule suppressions. So we will only consider the charmed meson loops here. The decay amplitudes

for $\psi(3770) \rightarrow VP$ corresponding to the diagrams in Fig. 3 read as

$$
\begin{aligned}
\mathcal{M}_a &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi DD} \varepsilon_\psi^\mu (q_{1\mu} - q_{3\mu})][-4if_{D^*DV}\varepsilon_{\rho\sigma\xi\tau} p_V^\rho \varepsilon_V^{*\sigma} q_1^\xi][2ig_{D^*DP}p_P\phi] \\
&\quad \times \frac{i}{q_1^2 - m_1^2} \frac{i(-g^{\tau\phi} + q_2^\tau q_2^\phi/m_2^2)}{q_2^2 - m_2^2} \frac{i}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2) ,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\mathcal{M}_b &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi D^*D} \varepsilon_{\alpha\beta\mu\nu} p_\psi^\alpha \varepsilon_\psi^\beta (q_3^\nu - q_1^\nu)][-ig_{DDV}\varepsilon_V^{*\rho}(q_{1\rho} + q_{2\rho})][ig_{D^*DP}p_P\phi] \\
&\quad \times \frac{i}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i(-g^{\mu\phi} + q_3^\mu q_3^\phi/m_3^2)}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2) ,
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\mathcal{M}_c &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi D^*D} \varepsilon_{\alpha\beta\mu\nu} p_\psi^\alpha \varepsilon_\psi^\beta (q_3^\nu - q_1^\nu)][-4if_{D^*DV}\varepsilon_{\rho\sigma\xi\tau} p_V^\rho \varepsilon_V^{*\sigma} q_1^\xi][-ig_{D^*D^*P}\varepsilon_{\phi\lambda\theta\kappa} p_P^\lambda q_2^\theta] \\
&\quad \times \frac{i}{q_1^2 - m_1^2} \frac{i(-g^{\tau\kappa} + q_2^\tau q_2^\kappa/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\mu\phi} + q_3^\mu q_3^\phi/m_3^2)}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2) ,
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\mathcal{M}_d &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi D^*D} \varepsilon_{\alpha\beta\mu\nu} p_\psi^\alpha \varepsilon_\psi^\beta (q_3^\nu - q_1^\nu)][2ig_{D^*D^*V}\varepsilon_V^{*\rho} q_{1\rho} g_{\sigma\tau} + 4if_{D^*D^*V}\varepsilon_V^{*\rho}(2p_{V\tau}g_{\rho\sigma} - p_{V\sigma}g_{\rho\tau})] \\
&\quad \times [2ig_{D^*DP}p_P\phi] \frac{i(-g^{\mu\sigma} + q_1^\mu q_1^\sigma/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\tau\phi} + q_2^\tau q_2^\phi/m_2^2)}{q_2^2 - m_2^2} \frac{i}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2) ,
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathcal{M}_e &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi D^*D^*} \varepsilon_\psi^\alpha (2q_{1\alpha}g_{\beta\mu} + q_{3\beta}g_{\alpha\mu} - q_{1\mu}g_{\alpha\beta})][-4if_{D^*DV}\varepsilon_{\rho\sigma\xi\tau} p_V^\rho \varepsilon_V^{*\sigma} q_1^\xi][ig_{D^*DP}p_P\phi] \\
&\quad \times \frac{i(-g^{\tau\beta} + q_1^\tau q_1^\beta/m_1^2)}{q_1^2 - m_1^2} \frac{i}{q_2^2 - m_2^2} \frac{i(-g^{\mu\phi} + q_3^\mu q_3^\phi/m_3^2)}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2) ,
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\mathcal{M}_f &= (i^3) \int \frac{d^4 q_2}{(2\pi)^4} [g_{\psi D^*D^*} \varepsilon_\psi^\alpha (2q_{1\alpha}g_{\beta\mu} + q_{3\beta}g_{\alpha\mu} - p_{1\mu}g_{\alpha\beta})] \\
&\quad \times [2ig_{D^*D^*V}\varepsilon_V^{*\rho} q_{1\rho} g_{\sigma\tau} + 4if_{D^*D^*V}\varepsilon_V^{*\rho}(2p_{V\tau}g_{\rho\sigma} - p_{V\sigma}g_{\rho\tau})][-ig_{D^*D^*P}\varepsilon_{\phi\lambda\theta\kappa} p_P^\lambda q_2^\theta] \\
&\quad \times \frac{i(-g^{\beta\sigma} + q_1^\beta q_1^\sigma/m_1^2)}{q_1^2 - m_1^2} \frac{i(-g^{\kappa\tau} + q_2^\kappa q_2^\tau/m_2^2)}{q_2^2 - m_2^2} \frac{i(-g^{\mu\phi} + q_3^\mu q_3^\phi/m_3^2)}{q_3^2 - m_3^2} \prod_i \mathcal{F}_i(m_i, q_i^2).
\end{aligned}
\tag{23}
$$

The only parameter of the long-distance part is the cut-off parameter α which can be determined by two ways. One is to adopt the same cutoff parameter $\alpha = 0.35$ as determined in the $\psi(3686)$ decays. The other is to determine α by the branching ratio $BR(\psi(3773) \rightarrow J/\psi\eta) = (9 \pm 4) \times 10^{-4}$ assuming that this process is also dominated by the IML with the same cutoff parameter.

III. NUMERICAL RESULTS

As discussed in Sec. II, the contributions from the EM and short-distance part have been fixed by their correlations with the $\psi(3686)$ decays. There are two parameters to be determined, i.e. the cut-off parameter α in the charmed meson loop and the relative phase angle θ of the meson loop amplitude to the short-distance part. To evaluate the contributions from the long-distance part, two ways are used to determine the cut-off parameter in the charmed meson loop. One way is to use the same cut-off parameter $\alpha = 0.35$ in $\psi(3686) \rightarrow VP$ based on the same topology at the quark level. Since there is no $c\bar{c}$ annihilation short-distance contribution and the EM transition is small in $\psi(3770) \rightarrow J/\psi\eta$, the long-distance charmed meson loop is dominant in this process. As a result, the central value $BR(\psi(3770) \rightarrow J/\psi\eta) = (9 \pm 4) \times 10^{-4}$ gives another cut-off parameter $\alpha = 0.73$. Such a difference indicates that the vertex form factor which takes care of the non-local effects may depend on the kinematics strongly as shown in Ref. [40]. Another source of the uncertainties arising from $\psi(3770) \rightarrow J/\psi\eta$ is the SU(3) flavor

TABLE I: The exclusive and total branching ratios of $\psi(3770) \to VP$ with the cutoff parameter $\alpha = 0.35$ fixed in the $\psi(3686)$ decays.

$BR(\psi(3770) \to VP)$	EM	short-distance	long-distance	tot($0°$)	tot($180°$)
$\rho\eta$	2.0×10^{-8}	0	1.47×10^{-6}	1.47×10^{-6}	1.47×10^{-6}
$\rho\eta'$	1.17×10^{-8}	0	6.15×10^{-7}	6.15×10^{-7}	6.15×10^{-7}
$\omega\pi^0$	3.74×10^{-8}	0	2.06×10^{-6}	2.06×10^{-6}	2.06×10^{-6}
$\phi\pi^0$	1.61×10^{-10}	0	0	0	0
$\rho^0\pi^0$	7.18×10^{-9}	6.86×10^{-5}	7.25×10^{-5}	7.84×10^{-5}	2.04×10^{-4}
$\rho\pi$	1.43×10^{-8}	2.06×10^{-4}	2.17×10^{-4}	2.35×10^{-4}	6.11×10^{-4}
$\omega\eta$	1.55×10^{-9}	4.65×10^{-5}	5.2×10^{-5}	5.42×10^{-5}	1.43×10^{-4}
$\omega\eta'$	1.27×10^{-9}	1.72×10^{-5}	2.21×10^{-5}	2.13×10^{-5}	5.73×10^{-5}
$\phi\eta$	2.92×10^{-9}	1.28×10^{-5}	1.4×10^{-5}	2.63×10^{-8}	5.37×10^{-5}
$\phi\eta'$	2.68×10^{-9}	2.72×10^{-5}	3.30×10^{-5}	2.74×10^{-7}	1.2×10^{-4}
$K^{*+}K^- + c.c.$	8.59×10^{-9}	1.09×10^{-4}	9.64×10^{-5}	4.35×10^{-5}	3.68×10^{-4}
$K^{*0}\bar{K}^0 + c.c.$	1.97×10^{-8}	1.09×10^{-4}	7.82×10^{-5}	2.39×10^{-5}	3.5×10^{-4}
total	1.20×10^{-7}	5.27×10^{-4}	5.17×10^{-4}	3.83×10^{-4}	1.71×10^{-3}

symmetry breaking. Note that the coupling for $\psi(3770)$ to $D_s\bar{D}_s$ meson pair may be different from that for $\psi(3770)D\bar{D}$. Here we use SU(3) breaking parameter 0.92 extracted from universal study of $\psi(3686)$ and J/ψ decays to take account this effect, i.e. $g_{\psi(3770)D_s\bar{D}_s}/g_{\psi(3770)D\bar{D}} = 0.92$ [33]. That is also the reason why the cutoff parameter fixed in $\psi(3770) \to J/\psi\eta$ channel is a little smaller than that in Ref.[24] which does not consider SU(3) breaking.

In Table I and Table II the branching ratios from different parts are listed. For the total branching ratios, we consider two phase angles, i.e. $\theta = 0°$ and $180°$, for each decay modes. It shows that the EM contributions are of order $10^{-8} \sim 10^{-9}$ which is negligibly small. Meanwhile, the short-distance and long-distance contributions are both typically about $10^{-4} \sim 10^{-5}$ for the isospin-conserved channels, while the long-distance contributions are $10^{-6} \sim 10^{-7}$ for the isospin-violated channels. So it is reasonable to ignore the EM contributions.

The following points can be learned from the numerical results in Table I and Table II:

- Since the EM interaction plays a minor role in $\psi(3770) \to VP$, no significant charge asymmetry can be seen in KK^* channel.

- The branching ratio of $\psi(3770) \to \omega\eta$ from the long-distance part contribution is larger than that of $\omega\eta'$. This is due to the larger $n\bar{n}$ component in η. The larger $s\bar{s}$ in η' also gives the larger long-distance part contribution for $\phi\eta'$ than that for $\phi\eta$.

- The branching ratios of the $\rho\pi$ channel are found to be larger than those of other channels for both α values. This is mainly because of the phase space difference. And this channel can be possibly measured by the current BES-III experiment.

- By summing over all the VP channels, the first scenario gives the total branching ratio in a range of $3.83 \times 10^{-4} \sim 1.71 \times 10^{-3}$. While the second scenario gives $3.38 \times 10^{-2} \sim 5.23 \times 10^{-2}$. These two results set up a range of the inclusive $\psi(3770) \to VP$ branching ratio, which seems to contribute to the $\psi(3770)$ non-$D\bar{D}$ decay significantly.

With the present experimental data, we can not determine the relative phase between the short-distance and long-distance parts, although the range of the branching ratios are comparable with the recent experimental data [4, 6]. So we plot the phase angle dependence of the summed branching ratio in Fig. 4. The band indicates the range of the summed branching ratio between $\alpha = 0.35$ and 0.73 as an estimate of the $\psi(3770)$ non-$D\bar{D}$ decays into VP, which can be investigated by the BES-III experiment.

TABLE II: The exclusive and total branching ratios of $\psi(3770) \to VP$ with $\alpha = 0.73$ which is fixed by $BR(\psi(3770) \to J/\psi\eta) = 9 \times 10^{-4}$.

$BR(\psi(3770) \to VP)$	EM	short-distance	long-distance	tot($0°$)	tot($180°$)
$\rho\eta$	2.0×10^{-8}	0	6.30×10^{-6}	6.30×10^{-6}	6.30×10^{-6}
$\rho\eta'$	1.17×10^{-8}	0	2.61×10^{-6}	2.61×10^{-6}	2.61×10^{-6}
$\omega\pi^0$	3.74×10^{-8}	0	8.89×10^{-6}	8.89×10^{-6}	8.89×10^{-6}
$\phi\pi^0$	1.61×10^{-10}	0	0	0	0
$\rho^0\pi^0$	7.18×10^{-9}	6.86×10^{-5}	5.60×10^{-3}	4.45×10^{-3}	6.88×10^{-3}
$\rho\pi$	1.43×10^{-8}	2.06×10^{-4}	1.68×10^{-2}	1.34×10^{-2}	2.06×10^{-2}
$\omega\eta$	1.55×10^{-9}	4.65×10^{-5}	3.99×10^{-3}	3.19×10^{-3}	4.87×10^{-3}
$\omega\eta'$	1.27×10^{-9}	1.72×10^{-5}	1.67×10^{-3}	1.35×10^{-3}	2.01×10^{-3}
$\phi\eta$	2.92×10^{-9}	1.28×10^{-5}	9.28×10^{-4}	7.23×10^{-4}	1.16×10^{-3}
$\phi\eta'$	2.68×10^{-9}	2.72×10^{-5}	2.16×10^{-3}	1.70×10^{-3}	2.67×10^{-3}
$K^{*+}K^- + c.c.$	8.59×10^{-9}	1.09×10^{-4}	8.44×10^{-3}	6.65×10^{-3}	1.05×10^{-2}
$K^{*0}\bar{K}^0 + c.c.$	1.97×10^{-8}	1.09×10^{-4}	8.5×10^{-3}	6.69×10^{-3}	1.05×10^{-2}
total	1.20×10^{-7}	5.27×10^{-4}	4.25×10^{-2}	3.38×10^{-2}	5.23×10^{-2}

IV. CONCLUSION

In this work, we investigate the $\psi(3770)$ non-$D\bar{D}$ decays into VP channels which are mainly through the short-distance part and the long-distance part. The long-distance part is described by the OZI evading intermediate meson loops according to an effective Lagrangian method. This calculation enriches and improves our previous work [24] and confirms the dominance of the IML contributions in $\psi(3770) \to VP$ via the dispersive part of the transition amplitude. We make predictions for the branching ratios of the VP decay channels benefiting from the available measurement of $\psi(3686) \to VP$ decays and $\psi(3770) \to J/\psi\eta$. These two experimental constraints provide a range of the $\psi(3770) \to VP$ decay branching ratios which can be studied by the BES-III experiment.

It is interesting to see that the IML contributions as a non-perturbative mechanism indeed account for some deficit for the $\psi(3770)$ non-$D\bar{D}$ decays. Such a mechanism is strongly correlated with the $\psi(3686)$ decays since both states are located in the vicinity of the $D\bar{D}$ open threshold. Thus, they will experience large open threshold effects via the IML. This makes it important to make a coherent study of these processes with the help of more accurate measurements from experiment. We anticipate that these results can help us establish the IML as an important non-perturbative dynamic mechanism in the charmonium decays and provide insights into the long-standing $\psi(3770)$ non-$D\bar{D}$ decay and "$\rho - \pi$ puzzle. Further theoretical studies of other channels such as $\psi(3770) \to VT$, SP, VS etc., are also needed as a prediction or test of the non-perturbative mechanism via the IML.

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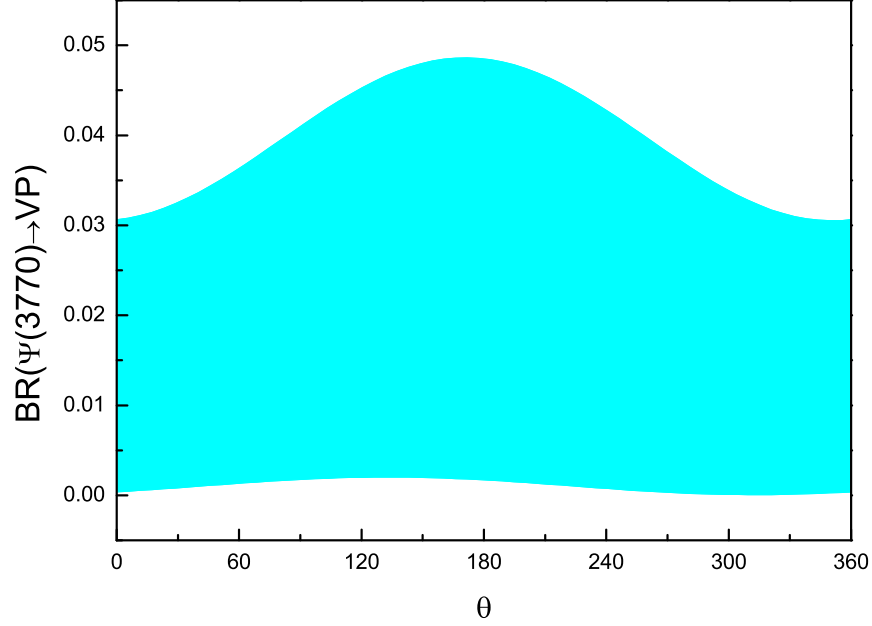
FIG. 4: The predicted total branching ratios of $\psi(3770) \to VP$ in terms of the relative phase θ between the short-distance and long-distance part. The lower limit is obtained with the cut-off parameter $\alpha = 0.35$ determined in $\psi(3686)$ decays, while the upper limit given by $\alpha = 0.73$ is fixed by the branching ratio $BR(\psi(3770) \to J/\psi\eta) = 9 \times 10^{-4}$.

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